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Exercises

**Section 8.1: 1(b), 2, 9, 10, 19, 20**

1. Using naive Gaussian elimination, factor the following matrices in the form A = LU, where L is a unit lower triangular matrix and U is an upper triangular matrix.

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Answer:

A equals space L U space equals space open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell l subscript 21 space 1 space space 0 space space 0 end cell row cell l subscript 31 space l subscript 32 space 1 space space 0 end cell row cell l subscript 41 space end subscript l subscript 42 space l subscript 43 space 1 end cell end table close square brackets open square brackets table row cell u subscript 11 space u subscript 12 space u subscript 13 space end subscript u subscript 14 end cell row cell 0 space space u subscript 22 space space u subscript 23 space u subscript 24 end cell row cell 0 space space 0 space space space space u subscript 33 space space space end subscript u subscript 34 space end cell row cell 0 space space 0 space space 0 space space u subscript 44 end cell end table close square brackets equals open square brackets table row cell 1 space space 0 space space 1 third space space 0 end cell row cell 0 space space 1 space space 3 space space minus 1 end cell row cell 3 space space minus 3 space space 0 space space 6 end cell row cell 0 space space 2 space space 4 space space minus 6 end cell end table close square brackets
u subscript 11 equals space 1 semicolon space u subscript 12 equals 0 semicolon space space u subscript 13 space end subscript space equals fraction numerator 1 space over denominator 3 end fraction semicolon u subscript 14 space equals space 0
l subscript 21 u subscript 11 space equals space 0 space minus greater than space l subscript 21 equals 0
l subscript 21 u subscript 12 space plus space u subscript 22 space equals space 1 space minus greater than space u subscript 22 space equals space 1
l subscript 21 u subscript 13 space plus space u subscript 23 space equals space 3 space minus space greater than space u subscript 23 space equals space 3
space l subscript 21 u subscript 14 space plus space u subscript 24 space equals space minus 1 space minus greater than space u subscript 24 space equals space minus 1
l subscript 31 u subscript 11 space equals space 3 space minus greater than space l subscript 31 space equals 3
l subscript 31 space u subscript 12 plus space l subscript 32 space u subscript 22 space equals space minus 3 space minus greater than space space l subscript 32 space equals space minus 3
l subscript 31 space u subscript 13 plus l subscript 32 space u subscript 23 space plus space u subscript 33 equals 0 space minus greater than space u subscript 33 equals space 8
l subscript 31 space u subscript 14 plus l subscript 32 space u subscript 24 space plus space u subscript 44 space equals space 6 space minus greater than space u subscript 34 space equals space 3
l subscript 41 space end subscript space equals space 0
l subscript 41 space end subscript u subscript 12 space plus space l subscript 42 space u subscript 22 space equals space 2 space minus greater than space l subscript 42 space equals space 2
l subscript 41 space end subscript u subscript 13 space plus space l subscript 42 space u subscript 23 space plus space space l subscript 43 u subscript 33 space equals 4 space minus greater than space space l subscript 43 equals negative 1 fourth
l subscript 41 space end subscript u subscript 14 space plus space l subscript 42 space u subscript 24 space plus space space l subscript 43 u subscript 34 space plus space u subscript 44 space equals negative 6 space minus greater than space l subscript 44 space equals space minus 13 over 4
L space equals space open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 3 space space minus 3 space space 1 space space 0 end cell row cell 0 space space 2 space space minus 1 fourth space space 1 end cell end table close square brackets semicolon space U space equals space open square brackets table row cell 1 space space 0 space space 1 third space space 0 end cell row cell 0 space space 1 space space 3 space space minus 1 end cell row cell 0 space space 0 space space 8 space space 3 end cell row cell 0 space space 0 space space 0 space space minus 13 over 4 end cell end table close square brackets

2. Consider the matrix

A picture containing table

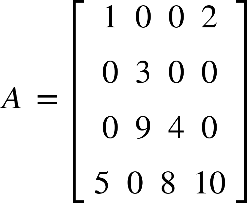
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a. Determine a unit lower triangular matrix M and an upper triangular matrix U such that MA = U.

b. Determine a unit lower triangular matrix L and an upper triangular matrix U such that A = LU. Show that M L = I so that L = M−1 .

Answer:

a. MA =U

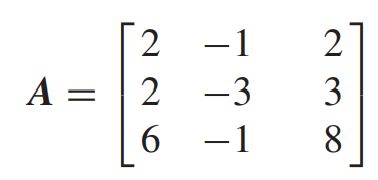


E subscript 1 equals open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space 0 space space 1 space space 0 end cell row cell negative 5 space space 0 space space 0 space space 1 end cell end table close square brackets
E subscript 2 equals open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 minus 3 space space 1 space space 0 end cell row cell 0 space space 0 space space 0 space space 1 end cell end table close square brackets
E subscript 3 equals open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space 0 space space 1 space space 0 end cell row cell 0 space space 0 space space minus 2 space space 1 end cell end table close square brackets
M space equals space E subscript 3 E subscript 2 E subscript 1 equals open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space minus 3 space space 1 space space 0 end cell row cell negative 5 space space 6 space space minus 2 space space 1 end cell end table close square brackets
U space equals space M times A space equals space open square brackets table row cell 1 space space 0 space space 0 space space 2 end cell row cell 0 space space 3 space space 0 space space 0 end cell row cell 0 space space 0 space space 4 space space 0 end cell row cell 0 space space 0 space space 0 space space 0 end cell row blank end table close square brackets

b. A= LU with L = M-1

A space equals L U space equals space open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space 3 space space 1 space space 0 end cell row cell 5 space space 0 space space 2 space space 1 end cell end table close square brackets open square brackets table row cell 1 space space 0 space space 0 space space 2 end cell row cell 0 space space 3 space space 0 space space 0 end cell row cell 0 space space 0 space space 4 space space 0 end cell row cell 0 space space 0 space space 0 space space 0 end cell end table close square brackets
M L space equals space space open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space minus space 3 space space 1 space space 0 end cell row cell negative 5 space space 6 space space minus 2 space space 1 end cell end table close square brackets space open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space 3 space space 1 space space 0 end cell row cell 5 space space 0 space space 2 space space 1 end cell end table close square brackets space equals open square brackets table row cell 1 space space 0 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 space space 0 end cell row cell 0 space space 0 space space 1 space space 0 end cell row cell 0 space space 0 space space 0 space space 1 end cell end table close square brackets equals I

9. Consider



a. Find the matrix factorization A = **LDU’** , where L is unit lower triangular, D is diagonal, and U’ is unit upper triangular.

b. Use this decomposition of A to solve Ax = b, where b = [−2, −5, 0]T .

Answer:

A space equals space open square brackets table row cell 2 space space minus 1 space space 2 end cell row cell 2 space space minus 3 space space 3 end cell row cell 6 space space minus 1 space space 8 end cell end table close square brackets
A space equals L U space equals space open square brackets table row cell 1 space space 0 space space 0 end cell row cell 1 space space 1 space space 0 end cell row cell 3 space space minus 1 space space 3 end cell end table close square brackets open square brackets table row cell 2 space space minus 1 space space 2 end cell row cell 0 space space minus 2 space space 1 end cell row cell 0 space space 0 space space 3 end cell end table close square brackets
A space equals space L D U apostrophe
U space equals space D U apostrophe space equals space open square brackets table row cell 2 space space 0 space space 0 end cell row cell 0 space space minus 2 space space 0 end cell row cell 0 space space 0 space space 3 end cell end table close square brackets open square brackets table row cell 1 space space minus 1 half space space 1 end cell row cell 0 space space 1 space space minus 1 half end cell row cell 0 space space 0 space space 1 end cell end table close square brackets space

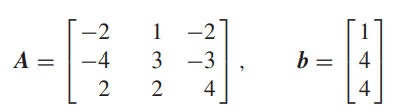
b.

A space equals space L D U apostrophe space semicolon space A x space equals space b
L D U apostrophe x space equals space b
P r o c e s s colon space L z space equals space b
open square brackets table row cell 1 space space 0 space space 0 end cell row cell 1 space space 1 space space 0 end cell row cell 3 space space minus 1 space space 1 end cell end table close square brackets open square brackets table row cell z subscript 1 end cell row cell z subscript 2 end cell row cell z subscript 3 end cell end table close square brackets equals open square brackets table row cell negative 2 end cell row cell negative 5 end cell row 0 end table close square brackets
T h e n space z subscript 1 equals space minus 2 semicolon space space z subscript 2 space equals space minus 3 semicolon space z subscript 3 space equals space 3
P r o c e s s colon space D y space equals space z
open square brackets table row cell 2 space space 0 space space 0 end cell row cell 0 space space minus 2 space space 0 end cell row cell 0 space space 0 space space 3 end cell end table close square brackets open square brackets table row cell y subscript 1 end cell row cell y subscript 2 end cell row cell y subscript 3 end cell end table close square brackets equals open square brackets table row cell negative 2 end cell row cell negative 3 end cell row 3 end table close square brackets
T h e n space space y subscript 1 equals space minus 1 semicolon space space y subscript 2 space equals space 3 over 2 semicolon space y subscript 3 space equals space 1
P r o c e s s colon space U apostrophe x space equals space y
open square brackets table row cell 1 space space fraction numerator negative 1 over denominator 2 end fraction space space 1 end cell row cell 0 space space 1 space space minus 1 half end cell row cell 0 space space 0 space space 1 end cell end table close square brackets open square brackets table row cell x subscript 1 end cell row cell x subscript 2 end cell row cell x subscript 3 end cell end table close square brackets equals open square brackets table row cell negative 1 end cell row cell 3 over 2 end cell row 1 end table close square brackets
T h e n space space x subscript 1 equals space minus 1 semicolon space space x subscript 2 space equals space 2 semicolon space x subscript 3 space equals space 1





10. Repeat the preceding problem for



Answer:

A space equals space L U space equals space open square brackets table row cell 1 space space 0 space space 0 end cell row cell 2 space space 1 space space 0 end cell row cell negative 1 space space 3 space space 1 end cell end table close square brackets open square brackets table row cell negative 2 space space 1 space space minus 2 end cell row cell 0 space space 1 space space 1 end cell row cell 0 space space 0 space space minus 1 end cell end table close square brackets
U space equals space D U apostrophe space equals space open square brackets table row cell negative 2 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 end cell row cell 0 space space 0 space space minus 1 end cell end table close square brackets open square brackets table row cell 1 space space minus 1 half space space 1 end cell row cell 0 space space 1 space space 1 end cell row cell 0 space space 0 space space 1 end cell end table close square brackets
L D U apostrophe x equals space b
P r o c e s s colon space L z space equals space b
open square brackets table row cell 1 space space 0 space space 0 end cell row cell 2 space space 1 space space 0 end cell row cell negative 1 space space 3 space space 1 end cell end table close square brackets open square brackets table row cell z subscript 1 end cell row cell z subscript 2 end cell row cell z subscript 3 end cell end table close square brackets equals open square brackets table row 1 row 4 row 4 end table close square brackets
T h e n space z subscript 1 space equals 1 space semicolon space z subscript 2 equals space 2 space semicolon space z subscript 3 space equals negative 1
P r o c e s s colon space D y space equals space z
open square brackets table row cell negative 2 space space 0 space space 0 end cell row cell 0 space space 1 space space 0 end cell row cell 0 space space 0 space space minus 1 end cell end table close square brackets open square brackets table row cell y subscript 1 end cell row cell y subscript 2 end cell row cell y subscript 3 end cell end table close square brackets equals open square brackets table row 1 row 2 row cell negative 1 end cell end table close square brackets
T h e n space y subscript 1 space equals negative fraction numerator 1 space over denominator 2 end fraction semicolon space y subscript 2 equals space 2 space semicolon space y subscript 3 space equals 1
P r o c e s s colon space U apostrophe x space equals space y
open square brackets table row cell 1 space space minus 1 half space space 1 end cell row cell 0 space space 1 space space 1 end cell row cell 0 space space 0 space space 1 end cell end table close square brackets open square brackets table row cell x subscript 1 end cell row cell x subscript 2 end cell row cell x subscript 3 end cell end table close square brackets equals open square brackets table row cell negative 1 half end cell row 2 row 1 end table close square brackets
T h e n space x subscript 1 space equals negative 1 semicolon space x subscript 2 equals space 1 space semicolon space x subscript 3 space equals 1



19. a. Prove that the product of two lower triangular matrices is lower triangular.

b. Prove that the product of two unit lower triangular matrices is unit lower triangular.

c. Prove that the inverse of a unit lower triangular matrix is unit lower triangular.

d. By using the transpose operation, prove that all of the preceding results are true for upper triangular matrices.

Answer:

1. Prove that the product of two lower triangular matrices is lower triangular.

Matrix A , matrix b, they have aij and bij entries, whenever j > i aij = 0 and bij = 0;, Let C = A\*BC subscript i j end subscript space equals space sum from k space equals space 1 to n of a subscript i k end subscript b subscript k j end subscript. space
W h e n e v e r space j space greater than space i comma space w e space h a v e colon space
C subscript i j end subscript space equals space sum from k space equals space 1 to i of a subscript i k end subscript b subscript k j end subscript space plus space sum from k space equals space i space plus 1 to n of a subscript i k end subscript b subscript k j end subscript space equals space sum from k space equals space 1 to i of a subscript i k end subscript times 0 space plus space sum from k space equals space i space plus 1 to n of 0 times b subscript k j end subscript space equals space 0 space
T h e n space m a t r i x space C space i s space a space l o w space t r i a n g l e space m a t r i x open parentheses P r o v e d close parentheses

1. Prove that the product of two unit lower triangular matrices is unit lower triangular .

Matrix A , matrix b, they have entries, aii = 1 and bii = 1;, Let C = A\*B

C subscript i i end subscript space equals space sum from k space equals space 1 to n of a subscript i k end subscript b subscript k i end subscript. space
C subscript i i end subscript space equals space sum from k space equals space 1 to i minus 1 of space a subscript i k end subscript b subscript k i end subscript space plus space a subscript i i end subscript b subscript i i end subscript space plus space sum from k space equals space i space plus 1 to n of a subscript i k end subscript b subscript k i end subscript space equals space sum from k space equals space 1 to i of a subscript i k end subscript times 0 space plus 1 plus space sum from k space equals space i space plus 1 to n of 0 times b subscript k i end subscript space equals space 1 space
T h e n space m a t r i x space C space i s space a space u n i t space l o w e r space t r i a n g u l a r open parentheses P r o v e d close parentheses

1. Prove that the inverse of a unit lower triangular. triangular matrix is unit lower

Matrix A is lower triangular matrix, and X, such AX = I. Consider column of X; AX(j) = I(j). Vector I subscript i superscript open parentheses j close parentheses end superscript space i s space 0 spaceexcept When i = j I subscript i superscript open parentheses j close parentheses end superscript space i s space 1 space

Since A is lower triangular matrix. AX(j) = I(j). Then

X subscript k superscript open parentheses j close parentheses end superscript space equals space 0 space w h e n space k space less than space j
W h e n space k space equals space j space
X subscript k superscript open parentheses j close parentheses end superscript space equals space fraction numerator I subscript i superscript open parentheses j close parentheses end superscript minus 0 over denominator a subscript j j end subscript end fraction equals fraction numerator space 1 over denominator 1 end fraction equals space 1
T h e n space X space open square brackets table row cell 1 space space 0 space space 0 space space space space midline horizontal ellipsis 0 end cell row cell space space space 1 space space 0 space space space space space space space space 0 end cell row cell space space space space space space space space space space 1 space space space space space space 0 space end cell row midline horizontal ellipsis row cell space space space space space space space space space space space space space space space 1 end cell end table close square brackets

1. By using the transpose operation, prove that all of the preceding results are true for upper triangular matrices.

Upper triangular matrices A and B , C = AB also be upper triangular since CT = AT BT is lower triangular since the transpose matrices AT and BT are lower triangular, and their product CT is once again lower triangular. Then its transpose C is upper triangular. Also, given unit upper triangular matrix A, its inverse A-1 is also unit upper triangular.

open parentheses A to the power of negative 1 end exponent A close parentheses to the power of T space equals space I to the power of T space equals space I
T h e n space A to the power of T open parentheses A to the power of negative 1 end exponent close parentheses to the power of T equals space open parentheses A to the power of negative 1 end exponent A close parentheses to the power of T space
I n v e r s e space A to the power of T space equals space open parentheses A to the power of negative 1 end exponent close parentheses to the power of T. space S o space open parentheses A to the power of T close parentheses to the power of negative 1 end exponent space equals space open parentheses A to the power of negative 1 end exponent close parentheses to the power of T space space space space

AT  is unit lover triangular and its invers open parentheses A to the power of T close parentheses to the power of negative 1 end exponent and open parentheses A to the power of negative 1 end exponent close parentheses to the power of T is lower triangular, Then A-1 is upper triangular. (proved)

20. Let L be lower triangular, U be upper triangular, and D be diagonal.

a. If **L** and **U** are both unit triangular and **LDU** is diagonal, does it follow that L and U are diagonal?

b. If **LDU** is nonsingular and diagonal, does it follow that L and U are diagonal?

c. If L and U are both unit triangular and if **LDU** is diagonal, does it follow that **L = U = I**?

Answer:

1. Consider D =0, then LDU = 0 regardless of what form L and U assume. L and U are not necessarily diagonal in this case.
2. If LDU is invertible, then D can be invertible since (LDU)-1 = U-1 D-1 L-1 and both L and U are invertible when they are unit triangular form and have nonzero determinant. Hence LDU to be invertible, must D mean its has no nonzero diagonal terms.

The product of a diagonal matrix with an unit upper triangular matrix is an upper triangular matrix:

DU = U’, since the diagonal matrix is just a row scaling matrix. We also know U’ is invertible when D has nonzero diagonal entries, U’ does not either. So U’ has a nonzero determinant and is invertible.

Consider: LDU = D’, we have L = D’U’-1. The inverse of an upper triangular matrix is a upper triangular, and also that the product of two upper triangular matrices is also upper triangular.

When U’-1 and D’ are upper triangular, Hence form of L is simultaneously upper triangular and lower triangular is for it to be diagonal.

From defining the lower triangular matrix L’ = LD, since the diagonal matrix is just a row scaling matrix. We also know L’ is invertible when D has nonzero diagonal entries, L’ does not either. So L’ has a nonzero determinant and is invertible.

Consider: LDU = D’, we have U = L’-1U’. The inverse of an upper triangular matrix is a lower triangular, and also that the product of two lower triangular matrices is also lower triangular.

When L’-1 and D’ are upper triangular, Hence form of U is simultaneously upper triangular and lower triangular is for it to be diagonal.

Hence, If LDU is nonsingular and diagonal, does it follow that L and U are diagonal.

1. Consider D =0, then LDU = 0 regardless of what form L and U assume.

If L and U are unit triangular and If LDU = I, does it follow that L = U = I?

Consider, if LDU is invertible, then both L and U are diagonal (from proved question b above). They are both unit triangular, it follow that L = U = I

**Section 8.2: 4, 7, 8, 11(a,c)**

4. (Multiple choice) From a vector norm, we can create a subordinate matrix norm. Which relation is satisfied by every subordinate matrix norm?

a. ||Ax|| ||A|| ||x||

b. ||I|| = 1

c. ||AB|| ||A|| ||B||

d. ||A+ B|| ||A||+||B||

e. None of these.

Answer: b. ||I|| = 1

7. (Multiple choice) A sufficient condition for the Jacobi method to converge for the linear system Ax = b.

a. A − I is diagonally dominant.

b. A is diagonally dominant.

c. G is nonsingular.

d. The spectral radius of G is less than 1.

e. None of these

Answer: b. A is diagonally dominant (strictly) , (d)

5,e, should be b;

6 should be c

9 should be c (No need to know)

8. (Multiple choice)

A sufficient condition for the Gauss-Seidel method to work on the linear system **Ax = b**.

a. A is diagonally dominant.

b. A − I is diagonally dominant.

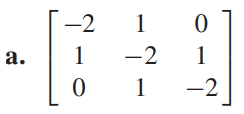
c. The spectral radius of A is less than 1.

d. G is nonsingular.

e. None of these

Answer: a. A is (strictly) diagonally dominant

11. Determine the condition numbers κ(A) of these matrices:



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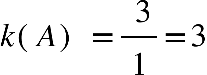
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Answer:

a.

A to the power of T A space equals space open square brackets table row cell negative 2 space space 1 space space 0 end cell row cell 1 space space minus 2 space space 1 end cell row cell 0 space space 1 space space minus 2 end cell end table close square brackets open square brackets table row cell negative 2 space space 1 space space 0 end cell row cell 1 space space minus 2 space space 1 end cell row cell 0 space space 1 space space minus 2 end cell end table close square brackets equals space open square brackets table row cell 5 space space minus 4 space space 1 end cell row cell negative 4 space space 6 space space minus 4 end cell row cell 1 space space minus 4 space space 5 end cell end table close square brackets
open square brackets table row cell open parentheses 5 minus lambda close parentheses space space minus 4 space space 1 end cell row cell negative 4 space space open parentheses 6 minus lambda close parentheses space space minus 4 end cell row cell 1 space space minus 4 space space open parentheses 5 minus lambda close parentheses end cell end table close square brackets equals 0
S o l v e space t h e space c h a r a c t e r i s t i c space p o l y n o m i a l space f o r space e i g e n v a l u e s space
open parentheses 5 minus lambda close parentheses open parentheses 6 minus lambda close parentheses open parentheses 5 minus lambda close parentheses space plus space 32 space minus space open parentheses 6 minus lambda close parentheses space minus space 32 open parentheses 5 minus lambda close parentheses space equals space 0
T h e n colon space lambda cubed space minus space 16 lambda squared space plus space 52 lambda space minus 16 space equals space 0
T h e n colon space open parentheses lambda minus 4 close parentheses open parentheses lambda squared minus 12 lambda plus 4 close parentheses equals 0
T h e n colon space lambda space equals space 4 semicolon space lambda space equals space 6 space plus-or-minus 4 square root of 2
k open parentheses A close parentheses square root of fraction numerator 6 space plus 4 square root of 2 over denominator 6 space minus 4 square root of 2 end fraction end root equals fraction numerator 6 space plus 4 square root of 2 over denominator 2 end fraction space almost equal to 5.83




c. Its singular values. 

**Section 8.3: 8, 9, 10, 11, 12, 13, 14, 15**

8. (Multiple choice) Let A be an n ×n invertible (nonsingular) matrix. Let x be a nonzero vector. Suppose that **Ax = λx**. Which equation does not follow from these hypotheses?

a. Ak x = λk x

b. λ−k x = (A−1)k x for k ≥0

c. p(A)x = p(λ)x for any polynomial p

~~d. A~~~~k~~ ~~x = (1 − λ)~~~~k~~ ~~x~~

e. None of these.

Answer: d. Ak x = (1 − λ)k x

9. (Multiple choice) For what values of s will the matrix I − svv∗ be unitary, where v is a column vector of unit length?

a. 0, 1

b. 0, 2

c. 1, 2

d. 0, √2

e. None of these.

Answer: b. 0, 2 (should be c UT U = I; U = I – SVV\*) V\* = V transport then congregate (VV\*)\* = (V\*)\* V\*

U\*U = I then U ^-1 =U \*

Then I = (I – SVV\*)\*(I – SVV\*)

= (I – SVV\*)(I-SVV\*) = I -SVV\* -SVV\* + S^2 V(V\*V) V\*

= I -2SVV\* + S^2 VV\* = I – (2S – S^2) VV\*

With (2S – S^2) = 0

10. (Multiple choice) Let **U and V** be unitary n × n matrices, possibly complex. Which conclusion is not justified?

~~a. U + V is unitary.~~

b. U∗ is unitary.

c. UV is unitary.

d. U −vv∗ is unitary when ||v|| = √2 and v is a column vector.

e. None of these.

Answer: a. U + V is unitary

11. (Multiple choice) Which assertion is true?

~~a. Every n × n matrix has n distinct (different) eigenvalues.~~

~~b. The eigenvalues of a real matrix are real.~~

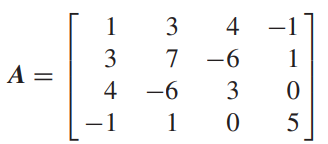
~~c. If U is a unitary matrix, then U∗ = U~~~~T~~

d. A square matrix and its transpose have the same eigenvalues.

~~e. None of these.~~

Answer: d. A square matrix and its transpose have the same eigenvalues

12. (Multiple choice) Consider the symmetric matrix



What is the smallest interval derived from Gershgorin’s Theorem such that all eigenvalues of the matrix A lie in that interval?

a. [−7, 9]

b. [−7, 13]

c. [3, 7]

d. [−3, 17]

e. None of these.

Answer: e. None of these (take the Union)

13. (True or false) Gershgorin’s Theorem asserts that every eigenvalue λ of an n ×n matrix A must satisfy one of these inequalities:

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Answer: True

14. (True or false) A consequence of Schur’s Theorem is that every square matrix A can be factored as A = PTP−1 , where P is a nonsingular matrix and T is upper triangular.

Answer: True

15. (True or false) A consequence of Schur’s Theorem is that every (real) symmetric matrix **A** can be factored in the form **A = PDP−1**, where **P** is unitary and **D** is diagonal.

Answer: True

Computing Exercises

**Section 8.1: 4, 8**

4. Write and test a procedure for determining A−1 for a given square matrix A of order n. Your procedure should use procedures Gauss and Solve.

Answer:

Source code:

clc

M = [0 2 3 2;

3 0 4 5;

0 5 0 2;

0 1 2 3];

A = M;

Ni = length(A(1,:));

Nj = length(A(:,1));

Position\_x = 1:Ni;

Position\_y = 1:Nj;

for index = 1: Ni-1

pct =index; qdt = index;

pivot\_checking = 0;

for i =index : Ni

for j = index:Ni

tmp = abs(A(uint8(Position\_x(i)),uint8(Position\_y(j))));

if (tmp > pivot\_checking)

pivot\_checking = tmp; pct = i; qdt = j;

end

end

end

if pivot\_checking == 0

fprintf("Pivot is zero, Can not inverse \n")

end

Position\_x([index pct]) = Position\_x([pct index]);

Position\_y([index qdt]) = Position\_y([qdt index]);

for i = index+1:Ni

if A(Position\_x(i),Position\_y(index)) ~= 0

mult = A(Position\_x(i),Position\_y(index))/A(Position\_x(index),Position\_y(index));

A(Position\_x(i),Position\_y(index)) = mult;

for j = index+1:Ni

A(Position\_x(i),Position\_y(j)) = A(Position\_x(i),Position\_y(j)) - mult\* A(Position\_x(index),Position\_y(j));

end

end

end

end

I = eye(size(A));

for index=1:Ni

for i = 2:Ni

for j =1:i-1

I(Position\_x(i),index) = I(Position\_x(i),index)-A(Position\_x(i),Position\_y(j))\*I(Position\_x(j),index);

end

end

end

Invert = zeros(Ni,Nj);

for index=1:Ni

for i = Ni:-1:1

for j =i +1:Ni

I(Position\_x(i),index) = I(Position\_x(i),index)-A(Position\_x(i),Position\_y(j))\*Invert(Position\_x(j),index);

end

Invert(Position\_y(i),index) = I(Position\_x(i),index)/A(Position\_x(i),Position\_y(i));

end

end

fprintf("\nOriginal Matrix:\n")

fprintf("%f %f %f %f \n",M);

fprintf("\nInverse Matrix:\n")

fprintf("%f %f %f %f \n",Invert);

Result:

Text

Description automatically generated

8. Investigate the numerical difficulties in inverting the following matrix:

A picture containing text

Description automatically generated

Answer:

**Code:**

A = [-0.0001, 5.096 , 5.101 , 1.853;

0.0 , 3.737 , 3.740 , 3.392;

0.0 , 0.0 , 0.006 , 5.254;

0.0 , 0.0 , 0.0 , 4.567];

B = inv(A)

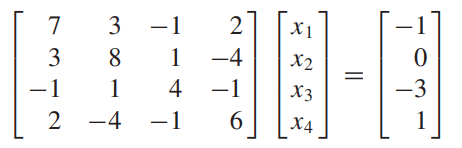
**Result:**

Table

Description automatically generated with medium confidence

**Section 8.2: 3, 4**

3. Using the Jacobi, Gauss-Seidel, and the SOR (ω = 1.4) iterative methods, write and run code to solve the following linear system to four decimal places of accuracy:



Compare the number of iterations in each case. Hint: Here, the exact solution is x = (−1, 1, −1, 1)T

Answer:

**Code for Using the Jacobi:**

clc;

a = [7 3 -1 2;

3 8 1 -4;

-1 1 4 -1;

2 -4 -1 6];

b = [-1 0 -3 1];

l\_0 =[0 0 0 0]';

T = 0.000001;

y = jacobi(a,b,l\_0,T);

fprintf ("Result Using the Jacobi y = %f %f %f %f \n",y)

function x1 = jacobi(a,b,l\_0,T)

n = length(b);

% Find value of x

for j = 1 : n

x(j) = ((b(j) - a(j,[1:j-1,j+1:n]) \* l\_0([1:j-1,j+1:n])) / a(j,j));

end

x1 = x';

k = 1;

while norm(x1-l\_0,1) > T

for j = 1 : n

temp(j) = ((b(j) - a(j,[1:j-1,j+1:n]) \* x1([1:j-1,j+1:n])) / a(j,j));

end

l\_0 = x1;

x1 = temp';

k = k + 1;

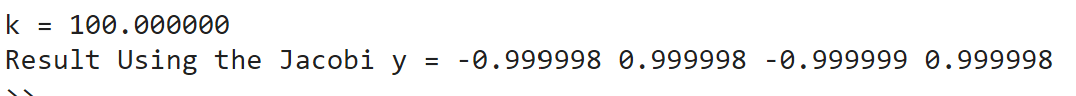
end

fprintf ("k = %f \n",k)

x = x1';

end

Result:



**Code Gauss-Seidel,:**

clc;

a = [7 3 -1 2;

3 8 1 -4;

-1 1 4 -1;

2 -4 -1 6];

b = [-1 0 -3 1];

l\_0=[0 0 0 0]';

T=1e-5;

y = GS(a,b,l\_0,T);

fprintf("Result Using the Gauss-Seidel y = %f %f %f %f \n",y)

%Display iteration

function x1 = GS(a,b,l\_0,T)

n=size(l\_0,1);

error=Inf;

% Assign values

k=0;

while error>T

l\_current=l\_0;

for i=1:n

sgma=0;

for j=1:i-1

sgma=sgma+a(i,j)\*l\_0(j);

end

for j=i+1:n

sgma=sgma+a(i,j)\*l\_current(j);

end

l\_0(i)=(1/a(i,i))\*(b(i)-sgma);

end

k=k+1;

error=norm(l\_current-l\_0);

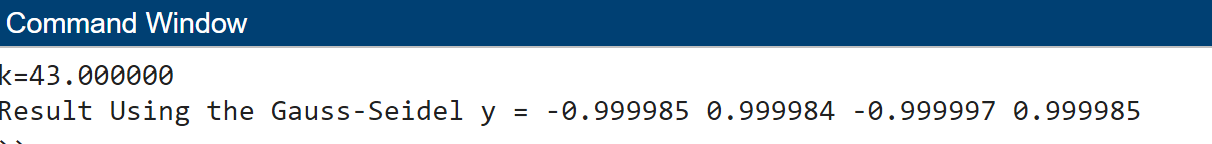
x1 = l\_0;

end

fprintf("k=%f \n",k)

end

**Result:**



**Code SOR (ω = 1.4)**

clc;

a = [7 3 -1 2;

3 8 1 -4;

-1 1 4 -1;

2 -4 -1 6];

b = [-1 0 -3 1];

l\_0=[0 0 0 0]';

T=1e-5;

y = SOR(a,b,l\_0,T);

fprintf("Result Using the SOR y = %f %f %f %f \n",y)

function x1 = SOR(a,b,l\_0,T)

lambda=1.4;

n=length(l\_0);

x=l\_0;

error=1;

k = 0;

while (error>T)

l\_current=x;

for i=1:n

I = [1:i-1 i+1:n];

x(i) = (1-lambda)\*x(i)+lambda/a(i,i)\*( b(i)-a(i,I)\*x(I) );

end

error = norm(x-l\_current)/norm(x);

k = k+1;

end

x1=x;

fprintf("k=%f \n",k);

end

**Result:**

Text

Description automatically generated with medium confidence

4. (Continuation) Solve the system using the SOR iterative method with values of ω = 1(0.1)2. Plot the number of iterations for convergence versus the values of ω. Which value of ω results in the fastest convergence?

Answer:

Code:

clear all;

clc;

a = [7 3 -1 2;

3 8 1 -4;

-1 1 4 -1;

2 -4 -1 6];

b = [-1 0 -3 1]';

l\_0=[0 0 0 0]';

T=1e-3;

list = [];

i=1;

MaxCon = 0;

current\_w = 0;

for lambda = 1:0.1:2

list(i) = SOR( a,b, l\_0, lambda, T );

% Finding fast convergence

if MaxCon < list(i)

MaxCon = list(i);

current\_lambda = lambda;

end

i= i+1;

end

fprintf("Fastest convergence at value of Lambda = %d", current\_lambda);

lambda = 1:0.1:2;

% Plot as follows

plot(list,lambda);

title("Plot the # of iterations for convergence versus ω");

xlabel("Number of list convergence ");

ylabel("Lambda values ");

function [list] = SOR(a, b,l\_0, lambda, T)

list = 0;

norm\_current = norm(b);

if (norm\_current == 0.0)

norm\_current = 1.0;

end

temp = b - a \* l\_0;

err = norm(temp) / norm\_current;

if (err < T)

return

end

b = lambda \* b;

M = lambda \* tril(a, -1) + diag(diag(a));

N = -lambda \* triu(a, 1) + (1.0 - lambda) \* diag(diag(a));

for list = 1 : (1/T)

x\_1 = l\_0;

l\_0 = M \ (N \* l\_0 + b); % adjust the aproximation

err = norm(x\_1 - l\_0, 1); % compute error

if (err <= T)

break

end

end

end

Result:

Text

Description automatically generated with medium confidence

Chart

Description automatically generated

**Section 8.3: 1, 10, 12**

1. Use Matlab, Maple, Mathematica, or other computer programs available to you to compute the eigenvalues and eigenvectors of these matrices:

A picture containing text

Description automatically generated

Answer:

Code:

clc;

A=[1 7;2 -5];

fprintf("Matrix A \n");

eigs(A)

B=[4 -7 3 2 3;1 6 11 -1 2;5 -5 -2 -4 1;9 -3 1 6 5;3 2 5 -5 1];

fprintf("Matrix B \n");

eigs(B)

Result:

Text, table

Description automatically generated

10. Using mathematical software such as Matlab, Maple, or Mathematica, compute the singular value decomposition of these matrices, and verify that each result satisfies the equation **A = UDVT** :

A picture containing graphical user interface

Description automatically generated

Create the diagonal matrix **D = UTAV** to check the results (always recommended). One can see the effects of roundoff errors in these calculations, for the off-diagonal elements in **D** are theoretically zero.

Answer:

a.

Code:

clc;

A = [1 1; 0 1;1 0];

%A = [1, 3,-2, 2, 7, 5, -2, -3, 4, 5, -3, -2];

[U, S, V] = svd(A)

U\*S\*V'

Result:

U =  
  
 -0.8165 -0.0000 -0.5774  
 -0.4082 -0.7071 0.5774  
 -0.4082 0.7071 0.5774  
  
  
S =  
  
 1.7321 0  
 0 1.0000  
 0 0  
  
  
V =  
  
 -0.7071 0.7071  
 -0.7071 -0.7071  
  
  
ans =  
  
 1.0000 1.0000  
 0.0000 1.0000  
 1.0000 -0.0000

b.

**Code:**

clc;

%A = [1 1; 0 1;1 0];

A = [1 3 -2; 2 7 5; -2 -3 4; 5 -3 -2];

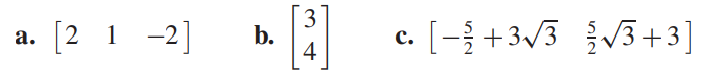
[U, S, V] = svd(A)

U\*S\*V'

**Result:**

U =  
  
 -0.1773 -0.4339 0.3176 -0.8243  
 -0.9125 -0.0655 -0.3961 0.0782  
 0.0778 0.7216 -0.4080 -0.5538  
 0.3603 -0.5354 -0.7588 -0.0880  
  
  
S =  
  
 9.4469 0 0  
 0 6.8963 0  
 0 0 4.7115  
 0 0 0  
  
  
V =  
  
 -0.0377 -0.6794 -0.7328  
 -0.8716 -0.3363 0.3567  
 -0.4888 0.6522 -0.5795  
  
  
ans =  
  
 1.0000 3.0000 -2.0000  
 2.0000 7.0000 5.0000  
 -2.0000 -3.0000 4.0000  
 5.0000 -3.0000 -2.0000

12. Find the singular value decomposition of these matrices:



A picture containing text, device, screenshot, gauge

Description automatically generated

Answer:

**Code:**

clc;

a = [2, 1, 2];

b = [3; 4];

c = [(-5/2)+3\*(3)^(1/2) (5/2)\*(3^(1/2))+3];

d = [2, 2, 2, 2; 17/10, 1/10, -17/10, -1/10; 3/5, 9/5, -3/5, -9/5];

e = [(7/2)-((13/6)\*6^(1/2)), (7/2)+((13/6)\*6^(1/2)); (-7/2)-((13/6)\*6^(1/2)), (-7/2)+((13/6)\*6^(1/2)); -((13/6)\*6^(1/2)), ((13/6)\*6^(1/2))] ;

fprintf("SVD of a is:\n ")

disp(svd(a))

[U, S, V] = svd(a)

fprintf("SVD of b is:\n ")

disp(svd(b))

[U, S, V] = svd(b)

fprintf("SVD of c is:\n ")

disp(svd(c))

[U, S, V] = svd(c)

fprintf("SVD of d is:\n ")

disp(svd(d))

[U, S, V] = svd(d)

fprintf("SVD of e is:\n ")

disp(svd(e))

[U, S, V] = svd(e)

Result:

SVD of a is:  
 3  
  
  
U =  
  
 1  
  
  
S =  
  
 3 0 0  
  
  
V =  
  
 0.6667 -0.3333 -0.6667  
 0.3333 0.9333 -0.1333  
 0.6667 -0.1333 0.7333  
  
SVD of b is:  
 5  
  
  
U =  
  
 0.6000 -0.8000  
 0.8000 0.6000  
  
  
S =  
  
 5  
 0  
  
  
V =  
  
 1  
  
SVD of c is:  
 7.8102  
  
  
U =  
  
 1  
  
  
S =  
  
 7.8102 0  
  
  
V =  
  
 0.3452 -0.9385  
 0.9385 0.3452  
  
SVD of d is:  
 4.0000  
 3.0000  
 2.0000  
  
  
U =  
  
 -1.0000 0 0  
 0 -0.6000 -0.8000  
 0 -0.8000 0.6000  
  
  
S =  
  
 4 0 0 0  
 0 3 0 0  
 0 0 2 0  
  
  
V =  
  
 -0.5000 -0.5000 -0.5000 -0.5000  
 -0.5000 -0.5000 0.5000 0.5000  
 -0.5000 0.5000 0.5000 -0.5000  
 -0.5000 0.5000 -0.5000 0.5000  
  
SVD of e is:  
 13.0000  
 7.0000  
  
  
U =  
  
 -0.5774 0.7071 -0.4082  
 -0.5774 -0.7071 -0.4082  
 -0.5774 -0.0000 0.8165  
  
  
S =  
  
 13.0000 0  
 0 7.0000  
 0 0  
  
  
V =  
  
 0.7071 0.7071  
 -0.7071 0.7071